



# Holyoke Public Schools

## Grade 7

### Variables and Patterns

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# Curriculum Maps Outline

## **Map Goals:**

1. To ensure that students are exposed to a rigorous curriculum in every school and every grade.
2. To have consistent instruction and assessment district wide.
3. To prepare students for the MCAS test.
4. To explain what is expected to be covered in each CMP or Investigations Unit.

## **Expectations:**

The district's expectation is for students to successfully meet the Massachusetts Mathematics Standards. In order to help facilitate this, teachers are required to follow the curriculum maps. The successful implementation of these maps requires teachers to thoroughly read each lesson in the TE and work through the project and problems in the map and the text prior to planning their lessons. Work should be kept in the binder with the curriculum map. Working through the math is an essential part of lesson planning, as it helps the teacher to better understand the concept being taught and the students' possible misunderstandings.

## **Feedback To Students:**

Feedback needs to happen daily in the classroom. There are many ways to give feedback. Conferencing, observations, questions asked during your opening, work time and closing are all forms of feedback.

## **Map Components:**

1. GENERAL PROBING QUESTIONS
2. UNIT SPECIFIC PROBING QUESTIONS
3. GOALS OF UNIT, CONTENT STANDARDS, & PERFORMANCE STANDARDS
4. PROJECT- to be done at end of unit and kept in the portfolio.
5. STUDENT MASTER – for project
6. INVESTIGATIONS
7. NOTEBOOK - includes: 3 Ring Binder, Bound Notebook, Portfolio
8. ACCOUNTABLE TALK – using probing questions
9. ON DEMAND ASSESSMENTS - to be done during teaching of unit.
10. STUDENT MASTERS- for on-demand assessments.

## Mathematics Evidence of Learning Artifacts

Artifact	K - 1	2 – 5	6 - 8
<b>3 Ring Binder</b>  <b>(3R)*</b>	<ul style="list-style-type: none"> <li>○ Student Work<sup>1</sup></li> </ul>	<ul style="list-style-type: none"> <li>○ Vocabulary</li> <li>○ Student sheets<sup>1</sup></li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>	<ul style="list-style-type: none"> <li>○ Math books</li> <li>○ Vocabulary</li> <li>○ Core Problems<sup>1</sup></li> <li>○ Lab sheets</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>
<b>Marble Notebook</b>  <b>(MNB)</b>	<ul style="list-style-type: none"> <li>○ Journal entries<sup>2</sup></li> </ul>	<ul style="list-style-type: none"> <li>○ Table of Contents</li> <li>○ Problem of the day</li> <li>○ Journal entries</li> <li>○ Class work</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation in the Table of Contents</u></b></p>	<ul style="list-style-type: none"> <li>○ Table of Contents</li> <li>○ Work time</li> <li>○ Journal entries</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation in the Table of Contents</u></b></p>
<b>Portfolio<sup>3</sup></b>  <b>(P)</b>	<ul style="list-style-type: none"> <li>○ On-demand tasks</li> <li>○ Projects</li> <li>○ Teacher anecdotal notes</li> </ul>	<ul style="list-style-type: none"> <li>○ On-demand tasks</li> <li>○ Reflections</li> <li>○ Projects</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>	<ul style="list-style-type: none"> <li>○ On-demand tasks</li> <li>○ Reflections</li> <li>○ Projects</li> </ul> <p style="text-align: center;"><b><u>All work should be dated and listed by investigation</u></b></p>

\* Folders may be used in place of binders for these grade levels

<sup>1</sup> Send home at the end of each unit

<sup>2</sup> Use grade level math journals

<sup>3</sup> All documents should be kept for the entire year

# Accountable Talk

## Probing Assessment Questions

As students progress through this unit, they should be asked the following questions to assess their specific knowledge of the unit.

- *What are the variables in the problem?*
- *How are these variables related to each other?*
- *Which variables depend on, or change in relation to, others?*
- *How can the relationship be displayed and analyzed with tables, graphs, and equations?*
- *What does it mean when we see regular and predictable change in a table of data or a graph?*
- *How can we use these predictable changes to solve problems?*

## Probing Questions – Teacher’s Role

The teacher’s role in probing for understanding is to ask questions that will:

- *Clarify student understanding*
- *Get at the objective of the lesson*
- *Go deeper into the mathematics*
- *Uncover misconceptions and misunderstandings*
- *Compare and contrast*

## **Probing Questions – Student’s Role**

The students’ role is to be an active participant by:

- *Explaining their strategies*
- *Asking clarifying questions to teacher and other students*
- *Being active listeners*
- *Using the language of mathematics*

## **Probing Questions - Suggestions**

When probing for understanding the teacher and students can use one or more of these suggested questions:

- *Why are you using  $<$   $>$ ?*
- *What are the ways you could  $<$   $>$ ?*
- *What else do you know?*
- *How do you know that?*
- *Can you show that?*
- *What convention did you use here?*
- *What can you do if you do not know?*
- *What standard does this work apply to?*
- *Is this always true?*
- *How does this connect to other mathematics we have learned?*
- *What is the same and what are the differences between  $<$   $>$ ?*
- *Can you back that up?*
- *Where is the math in your sketch?*
- *What does the answer mean?*
- *Does the answer make sense?*
- *Could you have used another operation to solve this task?*
- *Can you give examples?*

- *Can you say it another way?*
- *What's the math?*
- *Tell me about the task in your own words?*
- *What are you trying to find?*
- *How did you make your estimate?*
- *Will your answer be an over-estimate or an under-estimate? Why?*
- *I noticed that you used <....> to help you understand the task. Can you show us what you did and tell us how it helped you?*
- *Where do you see < > in your <model, diagram, number line, chart, etc.>?*
- *How can we see < > in your <model, diagram, number line, chart, etc.>?*
- *You have used a representation that is different from others that I've seen. Can you show us your <model, diagram, number line, chart, etc.>, and tell us how it helped you?*
- *How did you decide to solve the task? Why did you choose that method?*
- *Did you try any method that didn't work?*
- *Tell us what you tried.*
- *Why didn't it work?*
- *Would it ever work?*

# Goals, Content Standards, & Performance Standards

## Unit Goals:

- *Identify quantitative variables in situations.*
- *Recognize situations where changes in variables are related in useful patterns.*
- *Describe patterns of change shown in words, tables, and graphs of data.*
- *Construct tables and graphs to display relations among variables.*
- *Observe relationships between two quantitative variables as shown in a table, graph, or equation and describe how the relationship can be seen in each of the other forms of the representation.*
- *Use algebraic symbols to write rules and equations relating variables.*
- *Use tables, graphs, and equations to solve problems.*
- *Use graphing calculators to construct tables and graphs of relations between variables and to answer questions about these*

## Math Content Standards:

- *(7.P.1) Extend, represent, analyze, and generalize, a variety of patterns with tables, graphs, words, and, when possible, symbolic expressions. Include arithmetic and geometric progressions, e.g., compounding.*
- *(7.P.2) Evaluate simple algebraic expressions for given variable values, e.g.,  $3a^2 - b$  for  $a = 3$  and  $b = 7$ .*
- *(7.P.3) Create and use symbolic expressions and relate them to the verbal, tabular, and graphical representations.*
- *(7.P.5) Identify, describe, and analyze linear relationships between two variables. Compare positive rate of change, e.g.,  $y = 3x + 1$ , to negative rate of change, e.g.,  $y = -3x + 1$ .*
- *(7.P.6) Use linear equations to model and analyze problems involving proportional relationships. Use technology as appropriate.*
- *(7.G.1) Analyze, apply, and explain the relationship between the number of sides and the sums of the interior angle measures of polygons.*
- *(7.G.4) Graph points and identify coordinates of points on the Cartesian coordinate plane (all four quadrants).*

## **Performance Standards:**

- *(M3b) Represent relationships with tables, graphs, and verbal or symbolic rules.*
- *(M3c) Analyzes tables, graphs, and rules to determine functional relationships.*
- *(M4a) Collects data, organizes data, and displays data with tables, charts, and graphs that are appropriate, i.e., consistent with the nature of the data.*
- *(M4g) Formulates hypotheses to answer a question and uses data to test hypotheses.*

# Investigation 1: Variables, Tables, and Coordinate Graphs

<p><b><u>Objectives</u></b> Investigations 1.1 – 1.5</p>	<p><b><u>Pacing:</u></b> 7 days</p>
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u><sup>2:</sup></b> <a href="#">Appendix 2</a>, Investigation 1</p> <hr/> <p><b><u>Core Problems</u><sup>2:</sup></b> Variables and Patterns, Investigation 1 ACE Problems: # 1-3; 5, 26; 6-9, 27; 10, 12</p> <hr/> <p><b><u>Work Time</u><sup>1:</sup></b> Variables and Patterns, Problems 1.1 – 1.5</p> <hr/> <p><b><u>Journal Entries</u><sup>1:</sup></b> <a href="#">Appendix 3</a>, Inv 1.1 – 1.5</p> <hr/> <p><b><u>On Demand Tasks</u><sup>3:</sup></b> <a href="#">Appendix 5</a>, Investigation 1</p> <hr/> <p><b><u>Mathematical Reflection</u><sup>3</sup></b> <a href="#">Appendix 4</a>, MMR1:</p>
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>○ How do you decide which variable to put on which axis?</li> <li>○ What information can you find in a table? In a graph?</li> <li>○ What are some things to consider when making a scale for a graph?</li> <li>○ What are some advantages and disadvantages of tables, graphs, and written reports?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

1. *Marble Note Book*  
2.3 *Ring Binder*  
3. *Portfolio*

## Investigation 2: Analyzing Graphs and Tables

<p><b><u>Objectives</u></b> Investigations 2.1 – 2.3</p>	<p><b><u>Pacing:</u></b> 5 days</p>
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u><sup>2</sup>:</b> <a href="#">Appendix 2</a>, Investigation 2</p> <hr/> <p><b><u>Core Problems</u><sup>2</sup>:</b> Variables and Patterns , Investigation 2 ACE Problems: #3, 4, 16; 6, 22, 23; 8, 10-12</p> <hr/> <p><b><u>Work Time</u><sup>1</sup>:</b> Variables and Patterns, Problems 2.1 – 2.3</p> <hr/> <p><b><u>Journal Entries</u><sup>1</sup>:</b> <a href="#">Appendix 3</a>, Inv 2.1 – 2.3</p> <hr/> <p><b><u>On Demand Tasks</u><sup>3</sup>:</b> <a href="#">Appendix 5</a>, Investigation 2</p> <hr/> <p><b><u>Mathematical Reflection</u><sup>3</sup></b> <a href="#">Appendix 4</a>, MMR2:</p>
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>• How did you know which variable to put on which axis?</li> <li>• How did you know which variable was the independent variable? The dependent variable?</li> <li>• What are some advantages and disadvantages of tables, graphs, and written reports?</li> <li>• Did you try any method that didn't work?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

1. Marble Note Book  
2.3 Ring Binder  
3. Portfolio

## Investigation 3: Rules and Equations

<p><b><u>Objectives</u></b> Investigations 3.1 – 3.3</p>	<p><b><u>Pacing:</u></b> 5 days</p>
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u><sup>2</sup>:</b> <a href="#">Appendix 2</a>, Investigation 3</p>
	<p><b><u>Core Problems</u><sup>2</sup>:</b> Variables and Patterns, Investigation 3 ACE Problems: replace with pg and question #s</p>
	<p><b><u>Work Time</u><sup>1</sup>:</b> Variables and Patterns, Problems 3.1 - 3.3</p>
	<p><b><u>Journal Entries</u><sup>1</sup>:</b> <a href="#">Appendix 3</a>, Inv 3.1 – 3.3</p>
	<p><b><u>On Demand Tasks</u><sup>3</sup>:</b> <a href="#">Appendix 5</a>, Investigation 3</p>
<p><b><u>Mathematical Reflection</u><sup>3</sup></b> <a href="#">Appendix 4</a>, MMR3:</p>	
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>○ How did you pick your variables?</li> <li>○ Which variable is independent? Dependent?</li> <li>○ Which representation (table, graph, or equation) is most useful when?</li> <li>○ How do the patterns of change show up in the table, graph, and equation?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

1. Marble Note Book  
2.3 Ring Binder  
3. Portfolio

## Investigation 4: Calculator Tables and Graphs

<p><b><u>Objectives</u></b> Investigation 4.1 – 4.3</p>	<p><b><u>Pacing:</u></b> 5 day</p>
<p style="text-align: center;"><b><i>Evidence of Learning Artifacts</i></b></p> <p>Journal and Reflection questions should be posted and referred to at the beginning of the appropriate <i>Investigation</i>.</p> <p>Journal and Reflection entries need to be done in class as part of the closure and assessment</p>	<p><b><u>Vocabulary</u><sup>2</sup>:</b> <a href="#">Appendix 2</a>, Investigation 4</p> <p><b><u>Core Problems</u><sup>2</sup>:</b> Variables and Patterns, Investigation 4 ACE Problems: #2, 4, 17; 5, 6, 19</p> <p><b><u>Work Time</u><sup>1</sup>:</b> Variables and Patterns, Problem 4.1 – 4.3</p> <p><b><u>Journal Entries</u><sup>1</sup>:</b> <a href="#">Appendix 3</a>, Inv 4.1 – 4.3</p> <p><b><u>On Demand Tasks</u><sup>3</sup>:</b> <a href="#">Appendix 5</a>, Investigation 4</p> <p><b><u>Mathematical Reflection</u><sup>3</sup></b> <a href="#">Appendix 4</a>, MMR4:</p>
<p style="text-align: center;"><b><i>Accountable Talk</i></b></p> <p>To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</p>	<p>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:</p> <ul style="list-style-type: none"> <li>○ How did you pick your window size?</li> <li>○ How did you pick your scale?</li> <li>○ What does it mean when lines cross on a graph?</li> <li>○ What makes a graph useful?</li> <li>○ What makes a table useful?</li> </ul> <p>These are some recommended questions that you might use. Others can be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</p>

1. Marble Note Book  
2.3 Ring Binder  
3. Portfolio

# Appendix 1 Unit Project

<p><b><i>Project</i></b><sup>1</sup></p> <p>Student work should be placed in portfolio</p>	<p>The project is the culminating assessment, which will allow students to apply what they learned about the use of algebraic notation and reasoning. It is written in MCAS form to give students the experience of answering an open-response question.</p> <p>The unit project is called ‘Tile Pattern’ and the student handout for the project can be found in <a href="#">Appendix 1</a></p>
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*1. portfolio*

## Unit Project Scoring Guide

Score	Description
<u>4</u>	4 The response shows a comprehensive understanding of how to extend, represent and generalize a pattern with graphs and symbolic expressions.
<u>3</u>	3 The response shows a general understanding of how to extend, represent and generalize a pattern with graphs and symbolic expressions.
<u>2</u>	2 The response shows a basic understanding of patterns.
<u>1</u>	1 The response shows a minimal understanding of patterns.
<u>0</u>	0 The response is incorrect or contains some correct work that is irrelevant to the skill or concept being measured.

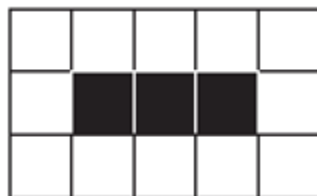
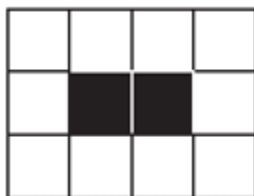
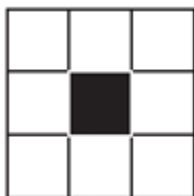
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DATE: \_\_\_\_\_

**PROJECT; Tile Pattern**

- BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.
- Show all work (diagrams, tables, and computations) on your answer sheet.
- If you do the work in your head, explain in writing how you did the work.

A worker placed white tiles around black tiles in the pattern shown in the three figures below.



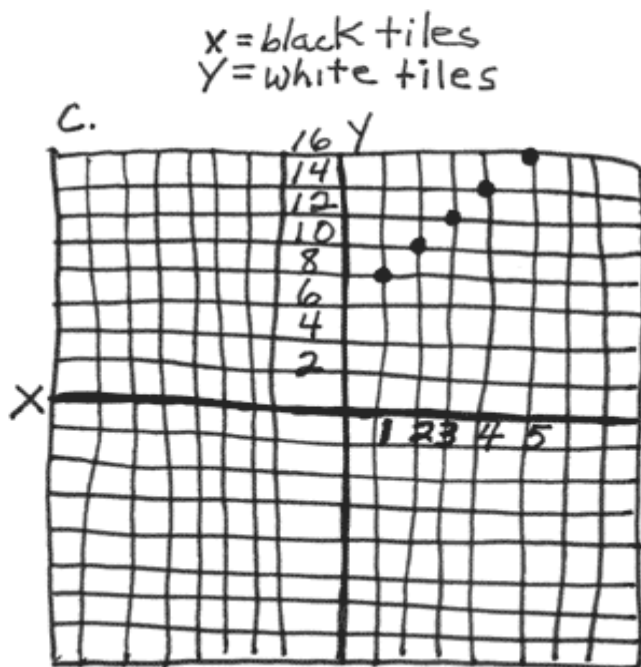
- a. Based on this pattern, how many white tiles would be needed for 4 black tiles?
  
- b. Based on this pattern, how many white tiles would be needed for 50 black tiles?
  
- c. Make a scatterplot of the first five figures in this pattern showing the relationship between the number of white tiles and the number of black tiles. Be sure to label the axes.
  
- d. Based on this pattern, explain how you could find the number of white tiles needed for any number,  $n$ , of black tiles. Show or explain your work.

# 2002 MCAS Grade 8 Mathematics

## Student Work: Question #9 - Score Point 4

- a. For a pattern with 4 black tiles you would need 14 white tiles.
- b. For a pattern with 50 black tiles you would need 106 white tiles.
- d. To find the number of white tiles for any number you could use the formula below

$$\begin{array}{ccc} & 2n+6 & \\ & / \quad \backslash & \\ 2(1)+6 & & 2(3)+6 \\ 8 & & 12 \end{array}$$

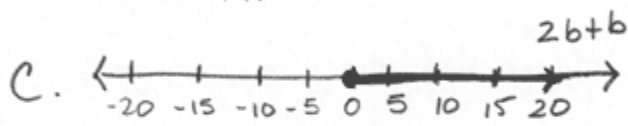


# 2002 MCAS Grade 8 Mathematics

## Student Work: Question #9 - Score Point 3

A. 14 white tiles would be needed for four black tiles.

B. 106 white tiles would be needed to make 50 black tiles



1 black tile = 8 white tiles

3 black tiles = 12 white tiles

2 black tiles = 10 white tiles

4 black tiles = 14 white tiles

5 black tiles = 16 white tiles

d.  $n = 2b + 6$   $B = \text{black}$ ,  $n = \text{white}$

It took a lot of thought to find this formula. I looked at the white and black tiles closely for quite a while. In all the patterns, there was 12 times the number of black tiles, plus 6. This formula works for every pattern of black and white tiles.

# 2002 MCAS Grade 8 Mathematics

## Student Work: Question #9 - Score Point 2

A= In my opinion, 6 white tiles would be needed for 4 black tiles.

B= There would have to be 100 white blocks.

D= In order to find the letter n, you have to add the number of black tiles to 7. And everytime you add another black block add 1 more to 7. Then you have to add 2 to the 7.

$$D = W = B + 7 + 2$$

$$W = 3 + 7 + 2$$

$$12 = 3 + 7 + 2$$

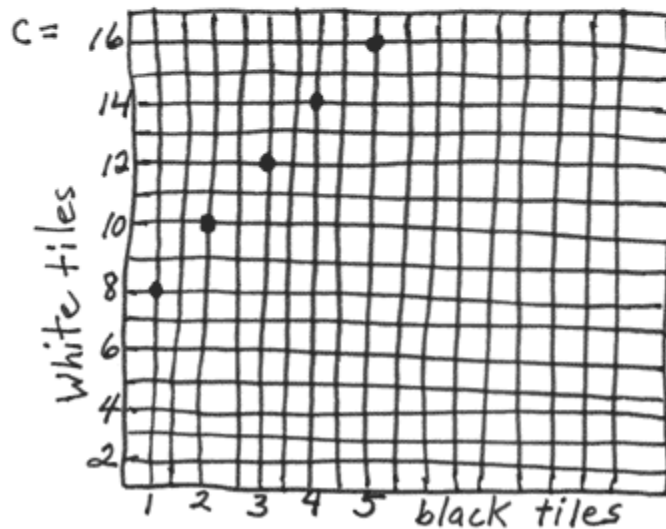
also

$$D = W = B + 8 + 2$$

$$W = 4 + 8 + 2$$

$$14 = 4 + 8 + 2$$

Everytime you add another black block you have to add another to the 7.

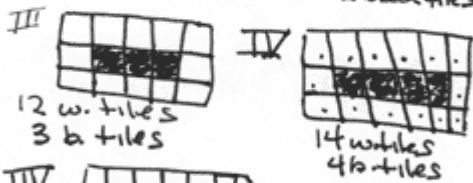
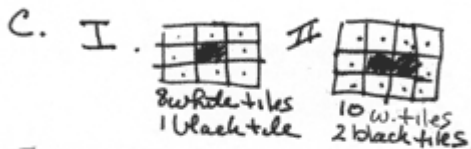


# 2002 MCAS Grade 8 Mathematics

## Student Work: Question #9 - Score Point 1

a. 12 white tiles would be needed for four black tiles.

b. 150 white tiles would be needed for 50 black tiles.



d. I could find out the # of tiles needed for every 1 black tile easily. For every one tile there are two other white tiles. For the first one you need to surround it with tiles. Then everytime you add a black one you add + two white ones.

# Appendix 2 Vocabulary

## **Investigation 1:**

- *relationship, variable, coordinate graph, dependent variable, independent variable, x-axis, y-axis, scale, coordinate pair*

## **Investigation 2:**

- *analyze*

## **Investigation 3:**

- *rule, equation, revenue, expense, profit*

## **Investigation 4:**

- *none*

# Appendix 3 Journal Entries

## **Investigation 1:**

### ***Investigation 1.1:***

How can you decide from your table if the jumper is slowing down or speeding up?

### ***Investigation 1.2:***

How can you decide from your graph if the jumper is slowing down or speeding up?

### ***Investigation 1.3:***

Sketch a coordinate graph showing temperature changing over the course of the day yesterday. Explain your sketch in words.

### ***Investigation 1.4:***

Give an example of a time when you should connect the points on a graph and a time when you should not. Explain.

### ***Investigation 1.5:***

How would riding into a strong wind affect the speed of the riders? How would riding with the wind at their backs affect their speed? What would each of these situations look like on a graph?.

## **Investigation 2:**

### ***Investigation 2.1***

Which bike shop should they choose if they need bikes for all the students in our class? Why?

### ***Investigation 2.2***

In your own words, describe how you decide which variable to put on the x-axis.

### ***Investigation 2.3***

Sketch a graph that shows something in your life changing over time. Explain what it shows.

## **Investigation 3**

### ***Investigation 3.1***

In the table, how do the distance entries for the three speeds compare? How do they show up in the graph?

### ***Investigation 3.2***

How is the equation for the bonus-card points similar to the equation for the admission price? How is it different?

### ***Investigation 3.3***

What strategy do you use to write an equation from a table?

## **Investigation 4**

### ***Investigation 4.1***

What are the advantages of using a graphing calculator?

### ***Investigation 4.2***

In Question B, what are the coordinates of the point where the line crosses the y-axis? What does this tell you about the bonus card situation?

### ***Investigation 4.3***

How does the table show where the costs are the same? How does the graph show where the costs are the same?

# Appendix 4 Reflections

## MMR1

How does the table show where the costs are the same? How does the graph show where the costs are the same?

## MMR2

Suppose the y-values increase as the x-values increase. How is the indicated in a table? In a graph?

Suppose the y-values decrease as the x-values increase. How is the indicated in a table? In a graph?

## MMR3

In this unit, you have represented relationships with tables, graphs, and equations. List some advantages and disadvantages of each of these representations.

## MMR4

What are the advantages and disadvantages of using tables and graphs to solve problems and search for patterns? In what ways is using a calculator more useful than using paper-and-pencil methods? In what ways is it less useful?

# Appendix 5 On Demand Tasks

## CMP2: Variables and Patterns

### *On-Demand Tasks*

#### *Additional Practice & Skills Workbook*

#### *Assessment Resources*

In class individualized On-Demand tasks assess knowledge of mathematical facts, operations, concepts, and skills, and their efficient application to problem solving. The results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.

#### *After Inv. 1*

[ACE Question #4 pg. 17]

#### *After Inv. 2*

[AP & S pg. 9 #2 a-d]

#### *After Inv. 3*

[ACE Questions 10, 12, 14, 16 pg. 57]

#### *After Inv. 4*

[ACE Question #1 pg. 72]

## HPS Mathematics Scoring Rubric

Score	Description
<u>4</u>	The response shows a <b><u>comprehensive</u></b> understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has <b><u>completed the task(s)</u></b> correctly, using mathematically sound procedures. It contains <b><u>clear, complete explanations</u></b> and/or <b><u>adequate work required</u></b> .
<u>3</u>	The response shows a <b><u>general</u></b> understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has <b><u>completed the task(s)</u></b> , using mathematically sound procedures. It contains <b><u>complete explanations</u></b> and/or <b><u>adequate work required</u></b> .
<u>2</u>	The response shows a <b><u>basic</u></b> understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It addresses <b><u>most aspects of the task(s)</u></b> , using mathematically sound procedures. It may contain a correct solution but provides <b><u>incomplete procedures, reasoning and/or explanations</u></b> . It may reflect <b><u>some misunderstandings</u></b> of the underlying mathematical concepts and/or procedures.
<u>1</u>	The response shows a <b><u>minimal</u></b> understanding of the mathematical concepts and/or procedures embodied in the task(s). It addresses <u>some elements of the task(s)</u> correctly but reaches an <b><u>inadequate solution and/or provides reasoning that is faulty or incomplete</u></b> . It exhibits <b><u>multiple flaws related to a misunderstanding of important aspects</u></b> of the task(s), misuse of mathematical procedures, or faulty mathematical reasoning. It reflects a <b><u>lack of essential understanding</u></b> of the underlying mathematical concepts. It may contain a correct numerical answer but <b><u>the required work is not provided</u></b> .
<u>0</u>	The response is <b><u>completely incorrect, irrelevant, or incoherent</u></b> , or contains a correct response arrived at using an <b><u>obviously incorrect procedure</u></b> .

**Investigation 1**

4. Katrina’s parents kept this record of her growth from her birth until her 18th birthday.

**Katrina’s Height**

Age (yr)	Height (in.)
birth	20
1	29
2	33.5
3	37
4	39.5
5	42
6	45.5
7	47
8	49
9	52
10	54
11	56.5
12	59
13	61
14	64
15	64
16	64
17	64.5
18	64.5



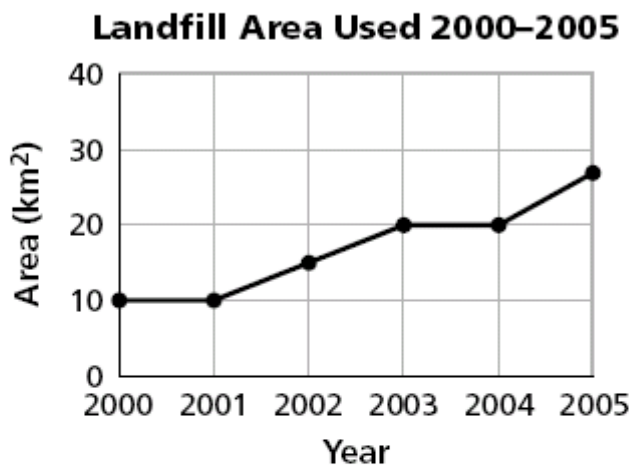
- a. Make a coordinate graph of Katrina’s height data.
- b. During which time interval(s) did Katrina have her greatest “growth spurt”?
- c. During which time interval(s) did Katrina’s height change the least?
- d. Would it make sense to connect the points on the graph? Why or why not?
- e. Is it easier to use the table or the graph to answer parts (b) and (c)? Explain.

**NAME:**

**DATE:**

**Investigation 2**

- *BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.*
  - *Show all work (diagrams, tables, and computations) on your answer sheet.*
  - *If you do the work in your head, explain in writing how you did the work.*
2. a. A newspaper included the graph below in a story about the amount of city land used for trash between 2000 and 2005. The graph shows the relationship between two variables. What are they?



- b. What is the difference between the least and greatest amount of land used for trash?
- c. Between which two years did the area used for trash stay the same?
- d. On this graph, what information is given by the lines connecting the points? Is this information necessarily accurate? Explain your reasoning.



NAME: \_\_\_\_\_

DATE: \_\_\_\_\_

### Investigation 4

1. Sean bought a DVD player and a receiver. The store offered him an interest-free payment plan with weekly installments. Sean figured out that after  $n$  weeks of payments, he would still owe  $175 - 7.5n$  dollars.
  - a. Make a calculator table showing what Sean owes after payments 1, 2, 3, and so on. Study the table to figure out the amount of his weekly payment. How is this amount shown in the table pattern?
  - b. Scan the table to find out how much Sean will owe after 20 weeks. Record the  $(x, y)$  table entry that shows the answer.
  - c. When will the amount Sean owes fall below \$100? Which  $(x, y)$  table entry shows the answer to this question?
  - d. When will Sean have paid for his items in full? How is this shown in the table?

# NOTES