Holyoke Public Schools

Grade 8

Looking for Pythagoras

Table of Contents

Curriculum Maps Outline

Map Goals:

1. To ensure that students are exposed to a rigorous curriculum in every school and every grade.
2. To have consistent instruction and assessment district wide.
3. To prepare students for the MCAS test.
4. To explain what is expected to be covered in each CMP or Investigations Unit.

Expectations:

The district's expectation is for students to successfully meet the Massachusetts Mathematics Standards. In order to help facilitate this, teachers are required to follow the curriculum maps. The successful implementation of these maps requires teachers to thoroughly read each lesson in the TE and work through the project and problems in the map and the text prior to planning their lessons. Work should be kept in the binder with the curriculum map. Working through the math is an essential part of lesson planning, as it helps the teacher to better understand the concept being taught and the students' possible misunderstandings.

Feedback To Students:

Feedback needs to happen daily in the classroom. There are many ways to give feedback. Conferencing, observations, questions asked during your opening, work time and closing are all forms of feedback.

Map Components:

1. GENERAL PROBING QUESTIONS
2. UNIT SPECIFIC PROBING QUESTIONS
3. GOALS OF UNIT, CONTENT STANDARDS, & PERFORMANCE STANDARDS
4. PROJECT- to be done at end of unit and kept in the portfolio.
5. STUDENT MASTER – for project
6. IVESTIGATIONS
7. NOTEBOOK - includes: 3 Ring Binder, Bound Notebook, Portfolio
8. ACCOUNTABLE TALK – using probing questions
9. ON DEMAND ASSESSMENTS - to be done during teaching of unit.
10. STUDENT MASTERS- for on-demand assessments.

Mathematics Evidence of Learning Artifacts

Artifact	K - 1	2 – 5	6 - 8
3 Ring Binder ***(3R)****	o Student Work[1]	o Vocabulary o Student sheets[1] ***<u>All work should be dated and listed by investigation</u>***	o Math books o Vocabulary o Core Problems[1] o Lab sheets ***<u>All work should be dated and listed by investigation</u>***
Marble Notebook ***(MNB)***	o Journal entries[2]	o Table of Contents o Problem of the day o Journal entries o Class work ***<u>All work should be dated and listed by investigation in the Table of Contents</u>***	o Table of Contents o Work time o Journal entries ***<u>All work should be dated and listed by investigation in the Table of Contents</u>***
Portfolio[3] ***(P)***	o On-demand tasks o Projects o Teacher anecdotal notes	o On-demand tasks o Reflections o Projects ***<u>All work should be dated and listed by investigation</u>***	o On-demand tasks o Reflections o Projects ***<u>All work should be dated and listed by investigation</u>***

[*] Folders may be used in place of binders for these grade levels
[1] Send home at the end of each unit
[2] Use grade level math journals
[3] All documents should be kept for the entire year

Accountable Talk

Probing Assessment Questions

As students progress through this unit, they should be asked the following questions to assess their specific knowledge of the unit.

Is it appropriate and useful to use the Pythagorean Theorem in this situation? How do I know this?

Do I need to find the distance between two points?

What are the quantities in this problem?

How can I estimate the square root of a number?

How can I find the length of something without directly measuring it?

Probing Questions – Teacher's Role

The teacher's role in probing for understanding is to ask questions that will:

- *Clarify student understanding*
- *Get at the objective of the lesson*
- *Go deeper into the mathematics*
- *Uncover misconceptions and misunderstandings*
- *Compare and contrast*

Probing Questions – Student's Role

The students' role is to be an active participant by:

- *Explaining their strategies*
- *Asking clarifying questions to teacher and other students*
- *Being active listeners*
- *Using the language of mathematics*

Probing Questions - Suggestions

When probing for understanding the teacher and students can use one or more of these suggested questions:

Why are you using <   >?

What are the ways you could <   >?

What else do you know?

How do you know that?

Can you show that?

What convention did you use here?

What can you do if you do not know?

What standard does this work apply to?

Is this always true?

How does this connect to other mathematics we have learned?

What is the same and what are the differences between <   >?

Can you back that up?

Where is the math in your sketch?

What does the answer mean?

Does the answer make sense?

Could you have used another operation to solve this task?

Can you give examples?

Can you say it another way?

What's the math?

Tell me about the task in your own words?

What are you trying to find?

How did you make your estimate?

Will your answer be an over-estimate or an under-estimate? Why?

I noticed that you used <….> to help you understand the task. Can you show us what you did and tell us how it helped you?

Where do you see <   > in your <model, diagram, number line, chart, etc.>?

How can we see <   > in your <model, diagram, number line, chart, etc.>?

You have used a representation that is different from others that I've seen. Can you show us your <model, diagram, number line, chart, etc.>, and tell us how it helped you?

How did you decide to solve the task? Why did you choose that method?

Did you try any method that didn't work?

Tell us what you tried.

Why didn't it work?

Would it ever work?

Goals, Content Standards, & Performance Standards

Unit Goals:

Relate the area of a square to the side length

Estimate the values of square roots of whole numbers

Locate irrational numbers on a number line.

Develop strategies for finding the distance between two points on a coordinate grid.

Understand the Pythagorean Theorem

Use the Pythagorean Theorem to solve everyday problems.

Math Content Standards:

8.N.2 – Define, compare, order, and apply frequently used irrational numbers, such as the square root of 2 and Pi.

8.N.3 – Use ratios and proportions in the solution of problems, in particular, problems involving unit rates, scale factors, and rate of change.

8.N.11 – Determine when an estimate rather than an exact answer is appropriate and apply in problem situations.

8.G.4 – Demonstrate an understanding of the Pythagorean theorem. Apply the theorem to the solution of problems.

8.G.5 – Use a straight¬edge, compass, or other tools to formulate and test conjectures, and to draw geometric figures.

8.G.7 – Identify three-dimensional figures (e.g., prisms, pyramids) by their physical appearance, distinguishing attributes, and spatial relationships such as parallel faces.

8.G.8 – Recognize and draw two-dimensional representations of three-dimensional objects, e.g., nets, projections, and perspective drawings.

8.M.1 – Select, convert (within the same system of measurement), and use appropriate units of measurement or scale.

8.M.2 – Given the formulas, convert from one system of measurement to another. Use technology as appropriate.

8.M.3 - Demonstrate an understanding of the concepts and apply formulas and procedures for determining measures, including those of area and perimeter/circumference of parallelograms, trapezoids, and circles. Given the formulas, determine the surface area and volume of rectangular prisms, cylinders, and spheres. Use technology as appropriate

Performance Standards:

M 1B - Uses and understands the inverse relationship between addition and subtraction, multiplication and division, and exponentiation and root-extraction; uses the inverse operation to determine unknown quantities in equations.

M 7A - Uses mathematical language and representations with appropriate accuracy, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs, and diagrams.

M 8E - Prepares a presentation or report that includes that question investigated, a detailed description of how the project was carried out, and an explanation of the findings.

M2D - Determines and understands length, area, and volume, including perimeter and surface are uses units, square units, and cubic units of measure correctly; computes areas of rectangles, triangles, and circles; computes volumes of prisms.

M2K - Models situations geometrically to formulate and solve problems

Investigation 1: Coordinate Grids

<table>
<tr><td colspan="1"><u>Objectives</u>
Investigations 1.1 – 1.3</td><td><u>Pacing:</u>
4 days</td></tr>
<tr><td rowspan="6">Evidence of Learning Artifacts

Journal and Reflection questions should be posted and referred to at the beginning of the appropriate Investigation.

Journal and Reflection entries need to be done in class as part of the closure and assessment</td><td><u>Vocabulary [2]:</u>
Appendix 2, Investigation 1</td></tr>
<tr><td><u>Core Problems [2]:</u>
Looking for Pythagoras, Investigation 1 ACE Problems: : [pgs. 12 – 14] # 1 – 4, 8, 10, 14, 15, 19, 20, 21, 24]</td></tr>
<tr><td><u>Work Time [1]:</u>
Looking for Pythagoras, Problems 1.1 – 1.3</td></tr>
<tr><td><u>Journal Entries [1]:</u>
Appendix 3, Inv 1.1 – 1.3</td></tr>
<tr><td><u>On Demand Tasks [3]:</u>
Appendix 5, Investigation 1</td></tr>
<tr><td><u>Mathematical Reflection [3]</u>
Appendix 4, MMR1:</td></tr>
<tr><td>Accountable Talk

To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</td><td>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:
o How did you know that…….?
o How can you use …?
o Can you show another way.to…….?
o What convention did you use?

These are some recommended questions that you might use. Others can be found be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</td></tr>
</table>

1.Marble Note Book
2.3 Ring Binder
3. Portfolio

Investigation 2: Squaring Off

Objectives Investigations 2.1 – 2.3	**Pacing:** 4 days

<table>
<tr><td rowspan="6">Evidence of Learning Artifacts

Journal and Reflection questions should be posted and referred to at the beginning of the appropriate Investigation.

Journal and Reflection entries need to be done in class as part of the closure and assessment</td><td><u>Vocabulary [2]:</u>
Appendix 2, Investigation 2</td></tr>
<tr><td><u>Core Problems [2]:</u>
Looking for Pythagoras , Investigation 2 ACE Problems: #1,2,42; 4-6,10,14-18; 35-37,41</td></tr>
<tr><td><u>Work Time [1]:</u>
Looking for Pythagoras, Problems 2.1 – 2.3</td></tr>
<tr><td><u>Journal Entries [1]:</u>
Appendix 3, Inv 2.1 – 2.3</td></tr>
<tr><td><u>On Demand Tasks [3]:</u>
Appendix 5, Investigation 2</td></tr>
<tr><td><u>Mathematical Reflection [3]</u>
Appendix 4, MMR2:</td></tr>
<tr><td>Accountable Talk

To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</td><td>As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:

How did you know…?
How can you justify…?
Does your answer make sense?
Can you draw me a diagram?

These are some recommended questions that you might use. Others can be found be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</td></tr>
</table>

1. Marble Note Book
2.3 Ring Binder
3. Portfolio

Investigation 3: The Pythagorean Theorem

<table>
<tr><td colspan="2"><u>Objectives</u>
Investigations 3.1 – 3.4</td><td><u>Pacing:</u>
6 days</td></tr>
<tr><td rowspan="6">Evidence of Learning Artifacts

Journal and Reflection questions should be posted and referred to at the beginning of the appropriate Investigation.

Journal and Reflection entries need to be done in class as part of the closure and assessment</td><td colspan="2"><u>Vocabulary [2]:</u>
Appendix 2, Investigation 3</td></tr>
<tr><td colspan="2"><u>Core Problems [2]:</u>
Looking for Pythagoras, Investigation 3 ACE Problems: #1,2,5,6,8-12; 23,26;24;15-17 pg 53-60</td></tr>
<tr><td colspan="2"><u>Work Time [1]:</u>
Looking for Pythagoras, Problems 3.1 – 3.4</td></tr>
<tr><td colspan="2"><u>Journal Entries [1]:</u>
Appendix 3, Inv 3.1 – 3.4</td></tr>
<tr><td colspan="2"><u>On Demand Tasks [3]:</u>
Appendix 5, Investigation 3</td></tr>
<tr><td colspan="2"><u>Mathematical Reflection [3]</u>
Appendix 4, MMR3:</td></tr>
<tr><td>Accountable Talk

To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</td><td colspan="2">As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:

What is your strategy?
How could you check that ….. is correct?
Could you begin with a different step?
Does …. make sense given the information in the problem?

These are some recommended questions that you might use. Others can be found be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</td></tr>
</table>

1. Marble Note Book
2.3 Ring Binder
3. Portfolio

Investigation 4: Using the Pythagorean Theorem

<table>
<tr><td><u>Objectives</u>
Investigations 4.1 – 4.4</td><td colspan="2"><u>Pacing:</u>
6 days</td></tr>
<tr><td rowspan="6">Evidence of Learning Artifacts

Journal and Reflection questions should be posted and referred to at the beginning of the appropriate Investigation.

Journal and Reflection entries need to be done in class as part of the closure and assessment</td><td colspan="2"><u>Vocabulary [2]:</u>
Appendix 2, Investigation 3</td></tr>
<tr><td colspan="2"><u>Core Problems [2]:</u>
Looking for Pythagoras, Investigation 3 ACE Problems: #1,2,5,6,8-12; 23,26;24;15-17 pg 53-60</td></tr>
<tr><td colspan="2"><u>Work Time [1]:</u>
Looking for Pythagoras, Problems 4.1 – 4.4</td></tr>
<tr><td colspan="2"><u>Journal Entries [1]:</u>
Appendix 3, Inv 4.1 – 4.4</td></tr>
<tr><td colspan="2"><u>On Demand Tasks [3]:</u>
Appendix 5, Investigation 3</td></tr>
<tr><td colspan="2"><u>Mathematical Reflection [3]</u>
Appendix 4, MMR3:</td></tr>
<tr><td>Accountable Talk

To promote learning, explore solutions, and justify reasoning, conversations between students and students or students and teacher must be accountable – accountable to the learning community, to the mathematics discipline, and to rigorous thinking.</td><td colspan="2">As a result of this Investigation, students should be able to talk and manipulate the vocabulary of the Investigation in response to this type of question:

Is there a way to write ….?
Does …. help you find the answer?
Does …. make sense given the information in the problem?

These are some recommended questions that you might use. Others can be found be found at the beginning of the map and on the probing question sheet in the district mathematics guide.</td></tr>
</table>

1. Marble Note Book
2.3 Ring Binder
3. Portfolio

Appendix 1 Unit Project

Project [1] Student work should be placed in portfolio	The project is the culminating assessment, which will allow students to apply what they learned about the use of the Pythagorean Theorem. It is written in MCAS form to give students the experience of answering an open-response question. The unit project is called 'Geometry" and the student handout for the project can be found in Appendix 1

1. portfolio

Unit Project Scoring Guide

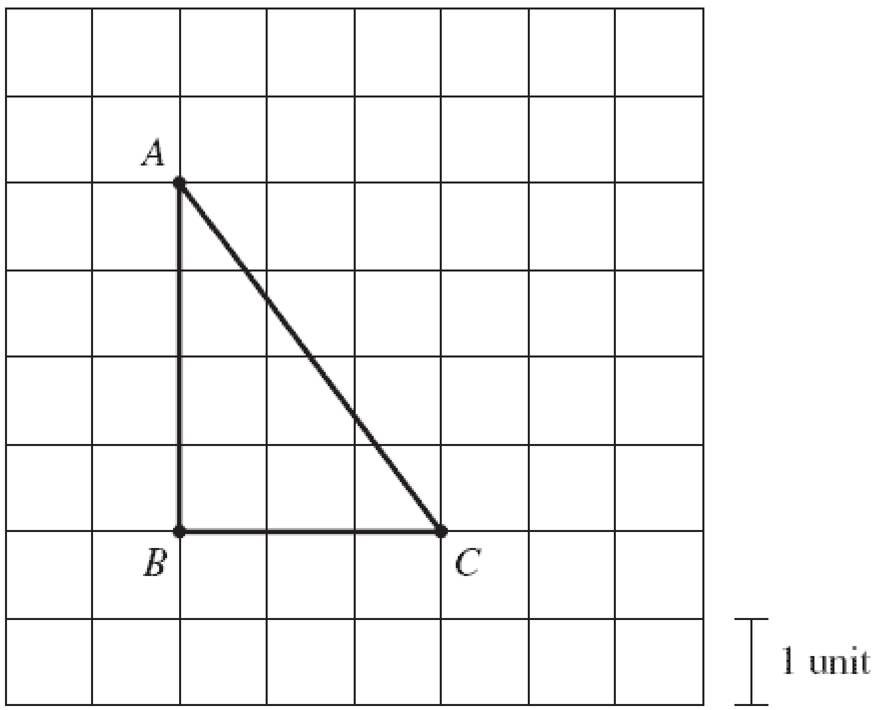
Score	Description
4	The student response demonstrates an exemplary understanding of the Geometry concepts involved in applying the Pythagorean theorem to the solutions of problems.
3	The student response demonstrates a good understanding of the Geometry concepts involved in applying the Pythagorean theorem to the solution of problems. Although there is significant evidence that the student is able to recognize and apply the concepts involved, some aspect of the response is flawed. As a result, the response merits 3 points.
2	The student response demonstrates a fair understanding of the Geometry concepts involved in applying the Pythagorean theorem to the solution of problems. While some aspects of the task are completed correctly, others are not. The mixed evidence provided by the student merits 2 points.
1	The student response demonstrates only a minimal understanding of the Geometry concepts involved in applying the Pythagorean theorem to the solution of problems.
0	The student response contains insufficient evidence of an understanding of the Geometry concepts involved in applying the Pythagorean theorem to the solution of problems to merit any points.

NAME: ___________________________ DATE: ___________

UNIT PROJECT: Geometry

- *BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.*
- *Show all work (diagrams, tables, and computations) on your answer sheet.*
- *If you do the work in your head, explain in writing how you did the work.*

The diagram below shows right triangle *ABC* drawn on a unit grid.



a. What is the length, in units, of line segment *AC*? Show or explain how you got your answer.
b. What is the area, in square units, of triangle *ABC*? Show or explain how you got your answer.
c. In your Student Answer Booklet, draw a rectangle that has the same area in square units as triangle *ABC*. Be sure to label the dimensions of your rectangle.

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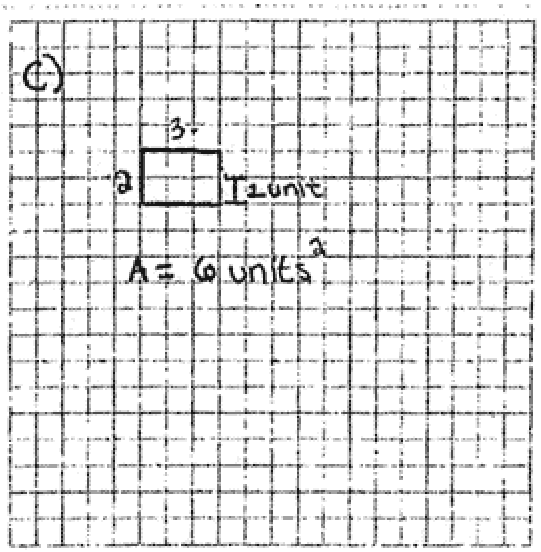
Student Work: Question #22 - Score Point 4

A) The length, in units, of the line segment AC is 5 units. I got my answer by using the Pythagorean Theorem. The length of line segment AB is 4 units, and the length of line segment BC is 3. I then calculated the formula $4^2 + 3^2 = c^2$; then I got $16 + 9 = c^2$; $16 + 9 = 25$ so $25 = c^2$. The square root of 25 is 5 so $c = 5$. And thats how I got 5 units for the length of the line segment AC.

B) The area in square units, of triangle ABC is 6 units². I got my answer by using the formula $A = \frac{1}{2}bh$ to find the area of the triangle. The base is 3 and the height is 4, so $3 \cdot 4$ equals 12. Half of 12 equals 6 so the area is 6 units².

C) see Graph



2005 MCAS Grade 8 Mathematics

Student Work: Question #22 - Score Point 3

A. The length of line segment AC is 25 units^2. To get this answer I used the Pythagorean Theorem. It states tha $a^2+b^2=c^2$. I used this and found the height and length of the triangle and squared them both. $4^2=16$ $16+9=25$
$3^2=9$

B. The area of the triangle is 6 units^2. When you multiply the base times the height then divide by two you will get the area.

$A=\frac{1}{2}b \cdot h$

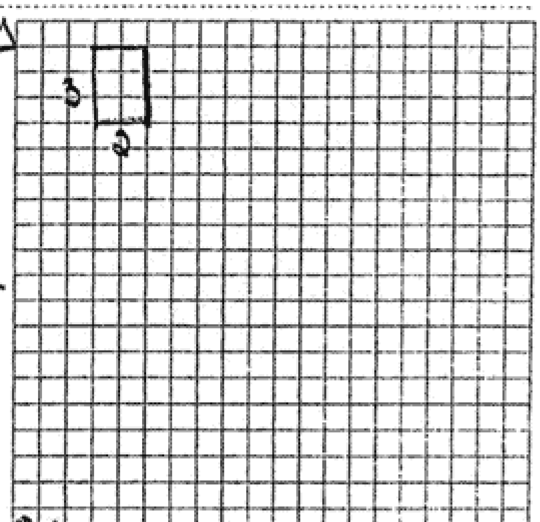
$A=\frac{1}{2}\,3 \cdot 4$

$A=\frac{1}{2}\,12$

$A=6$ units^2

C.

The rectangle I drew has an area of 6units^2 and so does the triangle. If the base and the height of the rectangle are the same as the triangle they are sure to both have the same area.



2005 MCAS Grade 8 Mathematics

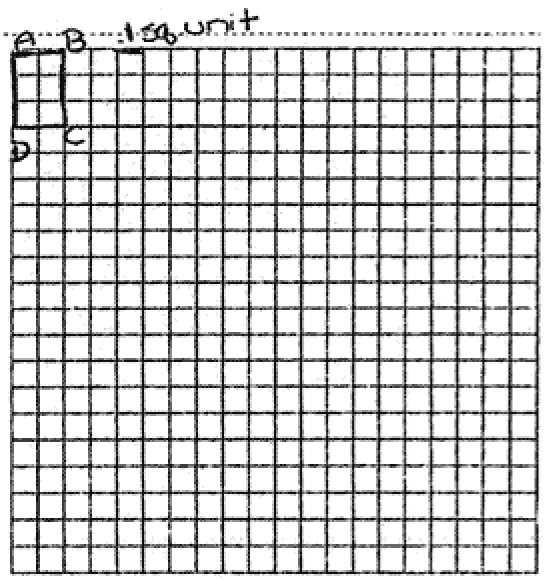
<u>Student Work: Question #22 - Score Point 2</u>

a) 3 units because the cutof is made up buy the other cut off

b) 6 square units
3 full
3 half full

c)

The Area of rectangle ABCD is 6 sq. units



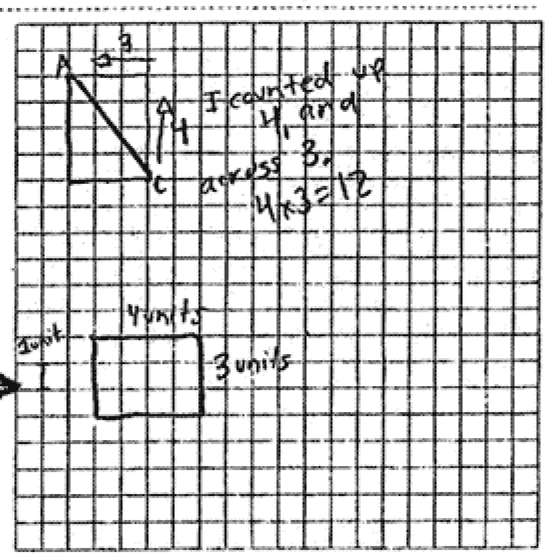
2005 MCAS Grade 8 Mathematics

Student Work: Question #22 - Score Point 1

a. The length is 4 units. I counted how many units the line passed through.

b. The area of triangle ABC is 12 units

c.



Appendix 2 Vocabulary

Investigation 1:

- *enclosure, subdivision, area, parallel, non - rectangle, square, parallelogram, right triangle*

Investigation 2:

- *square root, irrational, rational, integer, horizontal, vertica*

Investigation 3:

- *conjecture, hypotenuse, leg*

Investigation 4:

- *function, generate*

Appendix 3 Journal Entries

Investigation 1:

Investigation 1.1:

Will a direct helicopter route between two locations always be shorter than a car route? Explain your reasoning. [pg. 9 question E]

Investigation 1.2:

How can you differentiate between a square and a rectangle? [provide an example]

Investigation 1.3:

What is the difference between finding an area using subdivision, and finding an area using enclosure?

Investigation 2:

Investigation 2.1

What did you notice about the length of the squares sides as the squares area increased?

Why does this make sense?

Investigation 2.2

Explain the relationship between a square root and its corresponding square, using numbers, and pictures to justify your reasoning.

Investigation 2.3

Explain in detail one of the strategies you used to find the length of a line that is not horizontal or vertical.

Investigation 3

Investigation 3.1

Do you think the Pythagorean Theorem is true for all right triangles, even if the sides are not whole numbers? Justify your answer

Investigation 3.2

How can you use the Pythagorean Theorem to find the length of the hypotenuse of a right triangle?

Investigation 3.3

What are the benefits of using the Pythagorean theorem to find a length of a line segment?

Investigation 3.4

What statement can you make when the side lengths of a triangle does not satisfy $a^2 + b^2 = c^2$?

Why can you say that?

Investigation 4

Investigation 4.1

What two whole numbers would $\sqrt{17}$ fall between on a number line?

Investigation 4.2

What is the correct procedure for finding the hypotenuse of a figure given two leg lengths?

Investigation 4.3

What is the relationship of the length of the side opposite the hypotenuse and the hypotenuse's length?

Investigation 4.4

In Investigation 4.4, which triangles are similar and why?

Appendix 4 Reflections

MMR1

Describe some strategies you can use to find areas of figures drawn on dot paper. Give examples if it helps you explain your thinking. [pg. 18 # 3]

MMR2

Describe how you would find the length of a line segment connecting two dots on dot paper. Be sure to consider horizontal, vertical, and tilted segments. [pg. 30 # 1]

MMR3

How can you determine whether a triangle is a right triangle if you know only the lengths of its sides [pg. 45 # 3]

MMR4

Describe the special properties of a 30-60-90 triangle. [pg. 64 # 2]

Appendix 5 On Demand Tasks

CMP2: Looking for Pythagoras

On-Demand Tasks ***Additional Practice & Skills Workbook*** ***Assessment Resources*** In class individualized On-Demand tasks assess knowledge of mathematical facts, operations, concepts, and skills, and their efficient application to problem solving. The results of these different forms of assessment provide rich profiles of students' achievements in mathematics and serve as the basis for identifying curricula and instructional approaches to best develop their talents.	*After Inv. 1* Additional Practice and Skills Workbook #11-14 p29 NOTE: Each student will need 2 copies of this page in order to show both methods! *After Inv. 2* ACE Question #36 p 25 *After Inv. 3* Additional Practice and Skills Workbook #7-12 p32 *After Inv. 4* Additional Practice and Skills Workbook #9 p45

HPS Mathematics Scoring Rubric

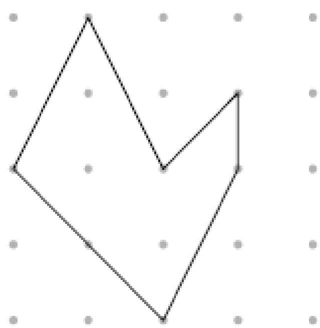
Score	Description
4	The response shows a **comprehensive** understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has **completed the task(s)** correctly, using mathematically sound procedures. It contains **clear, complete explanations** and/or **adequate work required**.
3	The response shows a **general** understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It indicates that the student has **completed the task(s)**, using mathematically sound procedures. It contains **complete explanations** and/or **adequate work required**.
2	The response shows a **basic** understanding of the mathematical concept(s) and/or procedures embodied in the task(s). It addresses **most aspects of the task(s)**, using mathematically sound procedures. It may contain a correct solution but provides **incomplete procedures, reasoning and/or explanations**. It may reflect **some misunderstandings** of the underlying mathematical concepts and/or procedures.
1	The response shows a **minimal** understanding of the mathematical concepts and/or procedures embodied in the task(s). It addresses some elements of the task(s) correctly but reaches an **inadequate solution and/or provides reasoning that is faulty or incomplete**. It exhibits **multiple flaws related to a misunderstanding of important aspects** of the task(s), misuse of mathematical procedures, or faulty mathematical reasoning. It reflects a **lack of essential understanding** of the underlying mathematical concepts. It may contain a correct numerical answer but **the required work is not provided**. .
0	The response is **completely incorrect, irrelevant, or incoherent**, or contains a correct response arrived at using an **obviously incorrect procedure**.

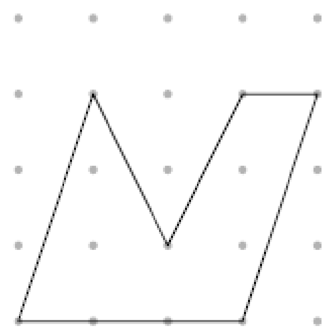
NAME: ______________________ DATE: __________

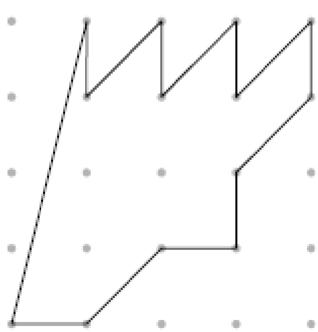
Investigation 1

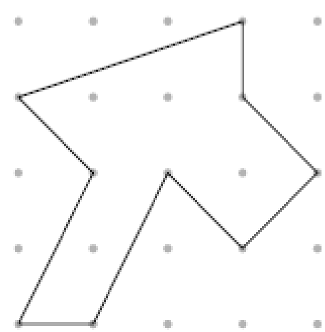
- *BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.*
- *Show all work (diagrams, tables, and computations) on your answer sheet.*
- *If you do the work in your head, explain in writing how you did the work.*

For questions #11 – 14, find the area of the figures 2ways (Subdivide and Enclose)

11.


12.


13.


14.


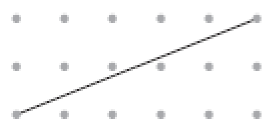
NAME: **DATE:**

Investigation 2

- *BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.*
- *Show all work (diagrams, tables, and computations) on your answer sheet.*
- *If you do the work in your head, explain in writing how you did the work.*

36. Consider this segment.

Web Co



a. Express the exact length of the segment, using the $\sqrt{\ }$ symbol.

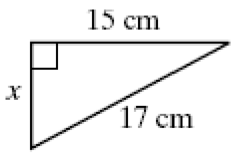
b. What two consecutive whole numbers is the length of the segment between?

NAME: **DATE:**

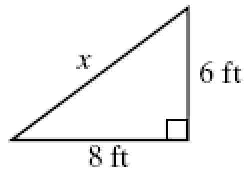
Investigation 3

Use the Pythagorean theorem to find the missing side of each right triangle.

7.



8.



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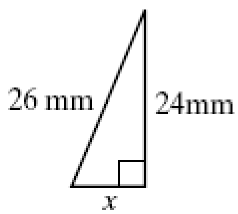

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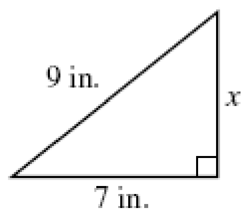

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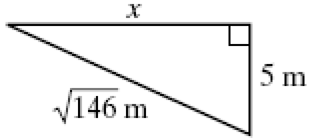

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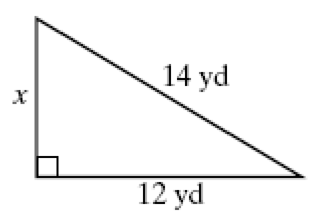
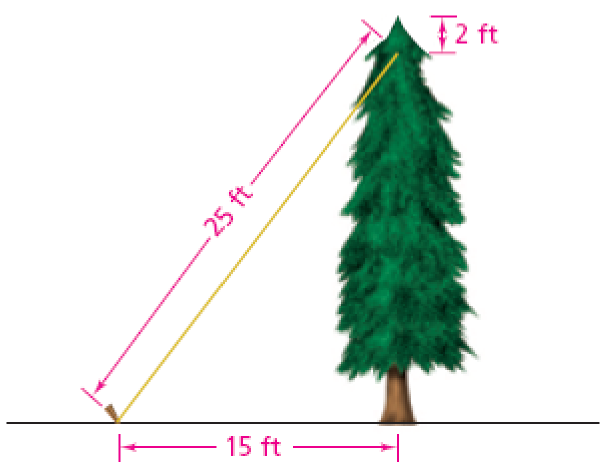


NAME: DATE:

Investigation 4

- *BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.*
- *Show all work (diagrams, tables, and computations) on your answer sheet.*
- *If you do the work in your head, explain in writing how you did the work.*

9. At Emmit's Evergreen Farm, the taller trees are braced by wires. A wire extends from 2 feet below the top of a tree to a stake in the ground. What is the tallest tree that can be braced with a 25-foot wire staked 15 feet from the base of the tree?



NOTES